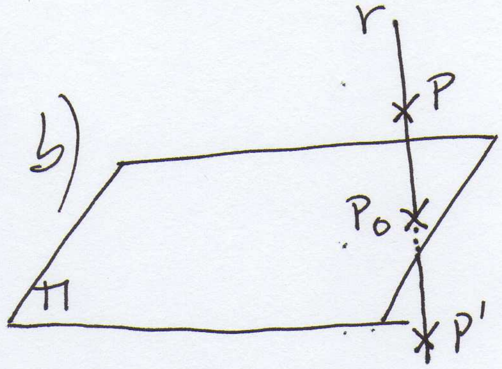
2007/1/A/4

$\Pi \equiv 2x+2y-z-6=0 \quad P(1,0,-1)$

a) $r \begin{cases} P \in r & P(1,0,-1) \\ r \perp \Pi \Rightarrow \vec{V_r} = \vec{N_\Pi} = (2,2,-1) \end{cases}$ $\boxed{\dfrac{x-1}{2} = \dfrac{y}{2} = \dfrac{z+1}{-1} \equiv r}$

b)



Para calcular el simétrico de P, P', hallemos en primer lugar P_0, intersección de r con Π:

$$r \equiv \frac{x-1}{2} = \frac{y}{2} = \frac{z+1}{-1} \Rightarrow \begin{cases} 2(x-1) = 2y; \; x-1 = y \; \boxed{x-y=+1} \\ -y = 2z+2; -2 = y+2z; \; \boxed{y+2z=-2} \end{cases}$$

El sistema sería: $\begin{cases} x-y = 1 \\ y+2z = -2 \\ 2x+2y-z = 6 \end{cases}$ $\quad x = 1+y; \; x = 1+(-2-2z) = -1-2z$
$y = -2-2z$

Hemos despejado en las dos primeras ecuaciones x e y para sustituirlas en la 3ª, en función de z:

$$2(-1-2z)+2(-2-2z)-z=6; \; -2-4z-4-4z-z=6;$$

$$-6-9z=6; \; -9z=12; \; z=\frac{12}{-9} = \boxed{-\frac{4}{3}=z}$$

$$x = -1-2\left(-\frac{4}{3}\right) = -1+\frac{8}{3} = \boxed{\frac{5}{3}=x} \qquad y = -2-2\left(-\frac{4}{3}\right) = -2+\frac{8}{3} = \frac{2}{3}$$

$$P_0 = \left(\frac{5}{3}, \frac{2}{3}, -\frac{4}{3}\right)$$

Como $\overrightarrow{PP_0} = \overrightarrow{P_0P'}$, con $P'(x,y,z)$

$$\left.\begin{array}{l} \overrightarrow{PP_0} = \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right) \\ \overrightarrow{P_0P'} = \left(x-\frac{5}{3}, y-\frac{2}{3}, z+\frac{4}{3}\right) \end{array}\right\} \begin{array}{l} \frac{2}{3} = x-\frac{5}{3}; \; x=\frac{7}{3} \\ \frac{2}{3} = y-\frac{2}{3}; \; y = \frac{4}{3} \\ -\frac{1}{3} = z+\frac{4}{3}; \; z = -\frac{5}{3} \end{array} \quad \boxed{P'\left(\frac{7}{3}, \frac{4}{3}, -\frac{5}{3}\right)}$$

2008/6/A/4.

$r \equiv mx = y = z+2 \quad (m \neq 0) \Rightarrow r \equiv \dfrac{x}{\frac{1}{m}} = \dfrac{y}{1} = \dfrac{z+2}{1}$

$s \equiv \dfrac{x-4}{4} = \dfrac{y-1}{1} = \dfrac{z}{2}$

a) m? $r \perp s \Rightarrow \vec{v_r} \perp \vec{v_s} \Rightarrow \vec{v_r} \cdot \vec{v_s} = 0 \Rightarrow$

$\left.\begin{array}{l} \vec{v_r} = \left(\frac{1}{m}, 1, 1\right) \\ \vec{v_s} = (4, 1, 2) \end{array}\right\} \vec{v_r} \cdot \vec{v_s} = \frac{1}{m} \cdot 4 + 1 + 2 = 0 \quad \frac{4}{m} = -3; \ \boxed{m = -\frac{4}{3}}$

b) Para r y s sean paralelas $\Rightarrow \vec{v_r} = k \cdot \vec{v_s} \Rightarrow$

$\left(\frac{1}{m}, 1, 1\right) = k(4, 1, 2) \Rightarrow \left\{\begin{array}{l} \frac{1}{m} = 4k \\ 1 = k \\ 1 = 2k; \ \underline{k = \frac{1}{2} \neq k = 1} \end{array}\right.$

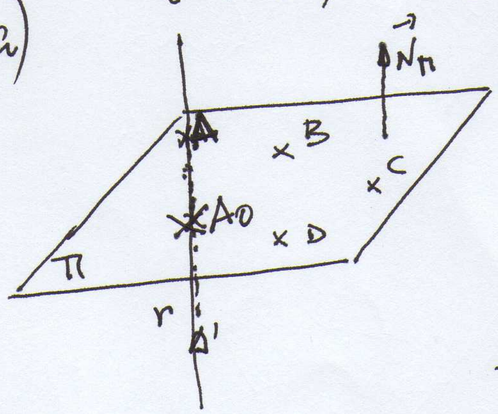
$\vec{v_r}$ y $\vec{v_s}$ no pueden ser nunca proporcionales, por tanto $\bar{r}$ y $\bar{s}$ no serán, nunca, paralelas.

—— x ——

2008/6/B/4.

$A(2,0,1) \quad B(-1,1,2) \quad C(2,2,1) \quad D(3,1,0)$

a)



$\left.\begin{array}{l} \vec{BC} = (3, 1, -1) \\ \vec{BD} = (4, 0, -2) \\ B(-1, 1, 2) \end{array}\right| \ \pi \equiv \begin{vmatrix} x+1 & y-1 & z-2 \\ 3 & 1 & -1 \\ 4 & 0 & -2 \end{vmatrix} = 0 \Rightarrow$

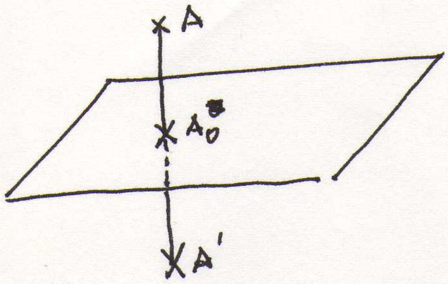
$\pi \equiv -2(x+1) - 4(y-1) - 4(z-2) + 6(y-1) = 0$

$\pi \equiv -2x - 2 - 4y + 4 - 4z + 8 + 6y - 6 = 0$

$\pi \equiv -2x + 2y - 4z + 4 = 0$

$\boxed{(:(-2)) \ \pi \equiv x - y + 2z - 2 = 0} \quad \vec{N_\pi}(1, -1, 2)$

b) $r \equiv \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{2}$ $A(2,0,1)$ $\vec{N}_\pi(1,-1,2)$



Hallamos A_0, proyección de A sobre π, calculando la intersección de r con π:

$$r \equiv \begin{cases} -(x-2) = y; & -x+2 = y; \quad x+y = 2 \\ 2y = -z+1; & 2y+z = 1 \end{cases}$$

$$\pi \equiv x - y + 2z = 2$$

El sistema sería: $\left.\begin{array}{ll} \text{I:} & x+y = 2 \\ \text{II:} & 2y+z = 1 \\ \text{III:} & x-y+2z = 2 \end{array}\right\}$ $\begin{array}{l} x+y = 2 \\ \text{III}-2\cdot\text{II}: \; x-5y = 0; \; x = 5y \end{array}$

$5y + y = 2; \quad y = \frac{2}{6} = \boxed{\frac{1}{3} = y}$

$x = 5y = 5\cdot\frac{1}{3}$ $\boxed{x = \frac{5}{3}}$

$\text{II}: \; 2\cdot\frac{1}{3} + z = 1; \; \boxed{z = \frac{1}{3}}$

Por tanto $A_0 = \left(\frac{5}{3}, \frac{1}{3}, \frac{1}{3}\right)$

Así $\overrightarrow{AA_0} = \overrightarrow{A_0A'}$. Si hacemos $A' = (x,y,z)$, tenemos:

$\overrightarrow{AA_0} = \left(\frac{5}{3}-2, \frac{1}{3}-0, \frac{1}{3}-1\right) = \left(-\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$

$\overrightarrow{A_0A'} = \left(x-\frac{5}{3}, y-\frac{1}{3}, z-\frac{1}{3}\right)$

$\left\{\begin{array}{l} x-\frac{5}{3} = -\frac{1}{3}; \; x = \frac{4}{3} \\ y-\frac{1}{3} = \frac{1}{3}; \; y = \frac{2}{3} \\ z-\frac{1}{3} = -\frac{2}{3}; \; z = -\frac{1}{3} \end{array}\right.$

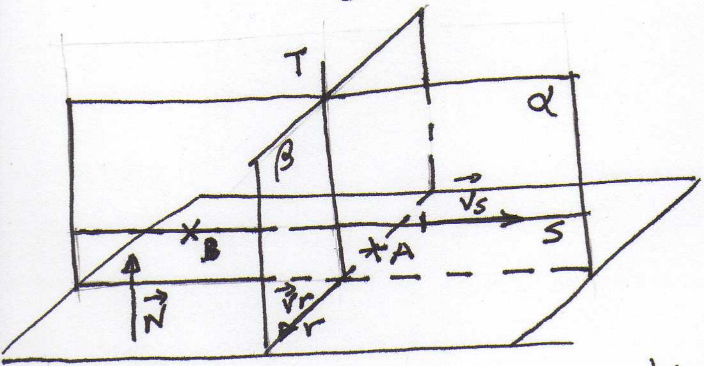
Por tanto, el simétrico de A respecto a π es

$\boxed{A'\left(\frac{4}{3}, \frac{2}{3}, -\frac{1}{3}\right)}$

2009/3/A/4

$$r \equiv \begin{cases} x = 1 \\ y = 1 \\ z = \lambda - 2 = -2 + \lambda \end{cases} \qquad \dot{A} \in r \Rightarrow \dot{A}(1,1,-2) \quad \vec{V_r} = (0,0,1)$$

$$s \equiv \begin{cases} x = \mu = \mu \\ y = \mu - 1 = -1 + \mu \\ z = -1 = -1 \end{cases} \qquad \dot{B} \in s \Rightarrow \dot{B}(0,-1,-1) \quad \vec{V_s} = (1,1,0)$$



$\vec{N} = \vec{V_r} \times \vec{V_s} \Rightarrow$ Vector $\perp$ a las 2 rectas

$$\vec{N} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \vec{j} - \vec{i} = -\vec{i} + \vec{j} = (-1,1,0)$$

La solución: T está formada por los planos α y β.

α estará determinado por $\bar{s}$ y $\vec{N} \Rightarrow \alpha \perp \bar{r}$

β " " por $\bar{r}$ y $\vec{N} \Rightarrow \beta \perp \bar{s}$

$$\alpha \Rightarrow \begin{cases} \dot{B}(0,-1,-1) \\ \vec{V_s} = (1,1,0) \\ \vec{N} = (-1,1,0) \end{cases} \qquad \alpha \equiv \begin{vmatrix} x & y+1 & z+1 \\ 1 & 1 & 0 \\ -1 & 1 & 0 \end{vmatrix} = z+1+z+1 = 0$$

$$\alpha \equiv 2z + 2 = 0; \quad (\div 2) \Rightarrow \boxed{\alpha \equiv z + 1 = 0}$$

$$\beta \Rightarrow \begin{cases} \dot{A}(1,1,-2) \\ \vec{V_r} = (0,0,1) \\ \vec{N} = (-1,1,0) \end{cases} \qquad \beta \equiv \begin{vmatrix} x-1 & y-1 & z+2 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix} = -(y-1) - (x-1) = -x - y + 2 = 0$$

$$\boxed{\beta \equiv x + y - 2 = 0}$$

$$\boxed{T \equiv \begin{cases} z + 1 = 0 \\ x + y - 2 = 0 \end{cases}}$$

2009/3/B/4

$$r \equiv \begin{cases} x+y=2 \\ y+z=0 \end{cases}$$

$$s \equiv \begin{cases} \dot{A}(2,1,0) \\ \dot{B}(1,0,-1) \end{cases} \quad \vec{v_s} = \vec{AB} = (-1,-1,-1) \rightarrow \vec{v_s} = (1,1,1)$$

$$\dot{B}(1,0,-1) \Rightarrow s \equiv \frac{x-1}{1} = \frac{y}{1} = \frac{z+1}{1} ; \begin{cases} x-1 = y ; & x-y=1 \\ y = z+1 ; & y-z=1 \end{cases}$$

a) Pos. relativa:

$$r \equiv \begin{cases} x+y \quad\quad = 2 & \mathrm{I} \\ \quad\; y+z = 0 & \mathrm{II} \end{cases} \qquad A/A^* = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right)$$

$$s \equiv \begin{cases} x-y \quad\quad = 1 & \mathrm{III} \\ \quad\; y-z = 1. & \mathrm{IV} \end{cases}$$

$$\text{rango } A: \quad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \quad \text{rango } A \geq 2$$

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1+1 \neq 0 \Rightarrow \text{rango } A = 3.$$

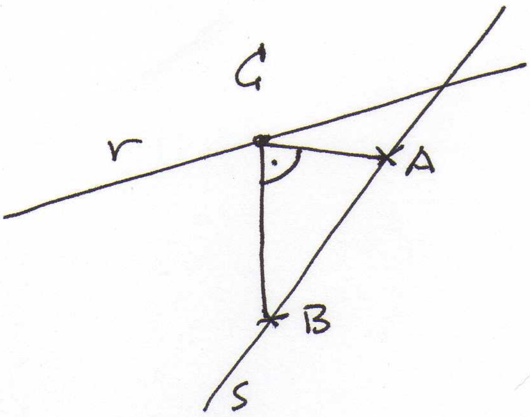
$$\text{rango } A^*: \quad A^* = \begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & 1 \end{vmatrix} \underset{F_3-F_1}{=} \begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & -1 \\ 0 & 1 & -1 & 1 \end{vmatrix} =$$

$$= +1 \begin{vmatrix} 1 & 1 & 0 \\ -2 & 0 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -1-1+2 = 0 \Rightarrow \text{rango } A^* = 3$$

r y s se cortan, el sist. de ec. es compatible deter.

Si sumamos las ecs. I y III: $2x = 3$; $x = 3/2$. Sustituimos en I: $y = 1/2$. Sustituimos en II $\frac{1}{2} + z = 0$; $z = -1/2$. Comprobamos en IV: $\frac{1}{2} - z = 1$; $z = \frac{1}{2} - 1 = -1/2$. Se cortan en el pto. $P(3/2, 1/2, -1/2)$ [ESTO NO SE PIDE EN EL PROBLEMA]

b) Calcula C en r tal que $\vec{CA}$ y $\vec{CB}$ sean perp.



$$r \equiv \begin{cases} x+y=2 \\ y+z=0 \end{cases}$$

Como y está en las 2 ecuaciones:

$$\left.\begin{aligned} x &= 2-y \\ y &= y \\ z &= -y \end{aligned}\right\}$$

Cualquier pto C de r tiene de coordenadas $C(2-y, y, -y)$

Por tanto, siendo $A(2,1,0)$ y $B(1,0,-1)$:

$\vec{CA}(y, 1-y, y)$ $\quad \vec{CB}(-1+y, -y, -1+y)$

$\vec{CA} \perp \vec{CB} \Rightarrow \vec{CA} \cdot \vec{CB} = 0 \Rightarrow y \cdot (-1+y) + (1-y) \cdot (-y) + y(-1+y) = 0$

$-y + y^2 - y + y^2 - y + y^2 = 0; \quad 3y^2 - 3y = 0;$

$$3y(y-1) = 0 \Rightarrow \begin{cases} 3y = 0; \ y = 0 \\ y-1 = 0; \ y = 1 \end{cases}$$

Hay dos soluciones para C: $C_1(2,0,0) \rightarrow y = 0$

$C_2(1,1,-1) \rightarrow y = 1.$